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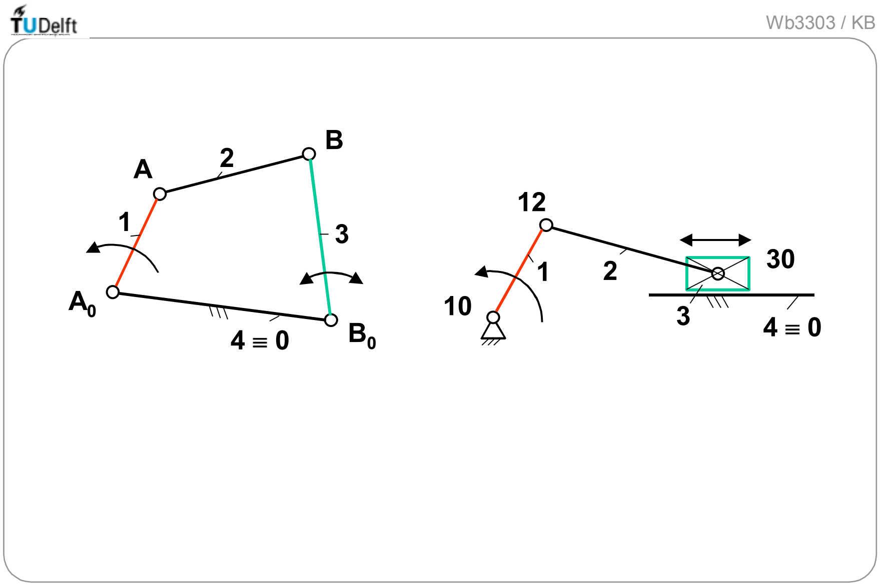

Fig. 2.1.1 Typical graphical representation of mechanisms, with indication of links, joints and input - output

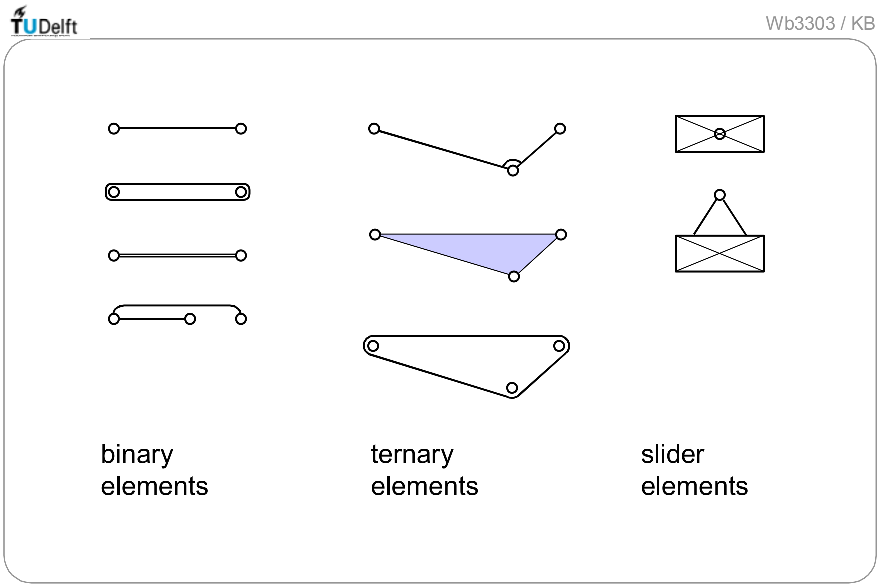

Fig. 2.1.2 Typical graphical representations of elements

2 Mechanism types

This chapter gives an impression of the numerous types of (planar) mechanism that can be used to generate non-uniform motion. Some rules will be presented that can be used to count the links, the connections and the degree of freedom. Attention is given to the systematic variation procedures to determine all possible mechanism types (structures). On the reverse a procedure will be explained to recognise the basic kinematic chain of a given mechanism. The most frequently used mechanism types and their characteristic motion behaviour will be highlighted.

2.1 Drawing symbols

A mechanism consists of a set of bodies (links, elements) and connections (joints). To make clear the structure of a mechanism a drawing is mostly helpful. In this book the graphical symbols as recommended in [2.1] will be used. A selection of these symbols is presented in figures 2.1.1 through 2.1.3. Some remarks to these symbols:

- For the purpose of describing the structure of planar mechanisms the links can often be represented simply by a line between two revolute joints.
- The sliding length is often immaterial, especially when the guiding element is the frame.
- In spatial mechanisms there is a difference between a prismatic joint (sliding without turning) and a cylindrical joint (sliding and turning), see figure 2.1.3. This difference does not exist for planar mechanisms.

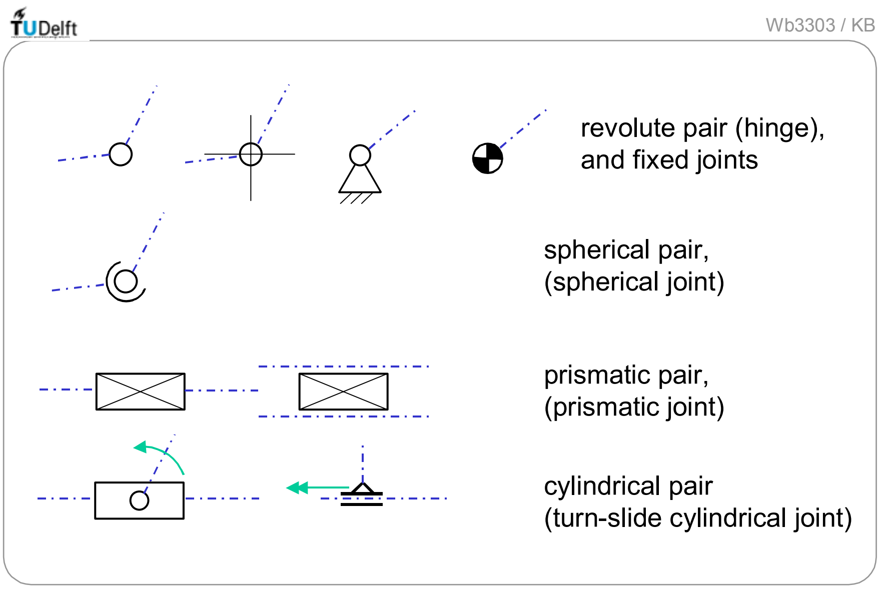

Fig. 2.1.3 Typical graphical representation of joints

2.2 The basic chain and Grübler's formula

A first idea about mechanism structure was already presented in chapter 1, figure 1.1.1 (repeated here as figure 2.2.1). Kinematic chains can be created at will by adding links, theoretically without any limit.

A planar chain with only revolute joints is called a *basic chain*. The attention is focussed first at such basic chains. In chapter 2.4 other joint types will be introduced and they will be called the topological variants of the basic chain.

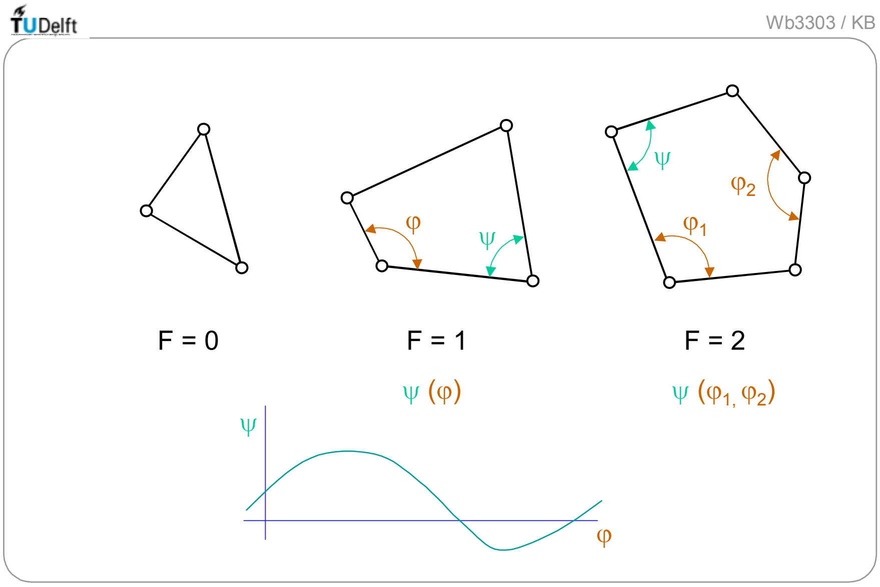

Fig. 2.2.1 Mechanisms and Degree of Freedom F (mobility)
Kinematic transfer function

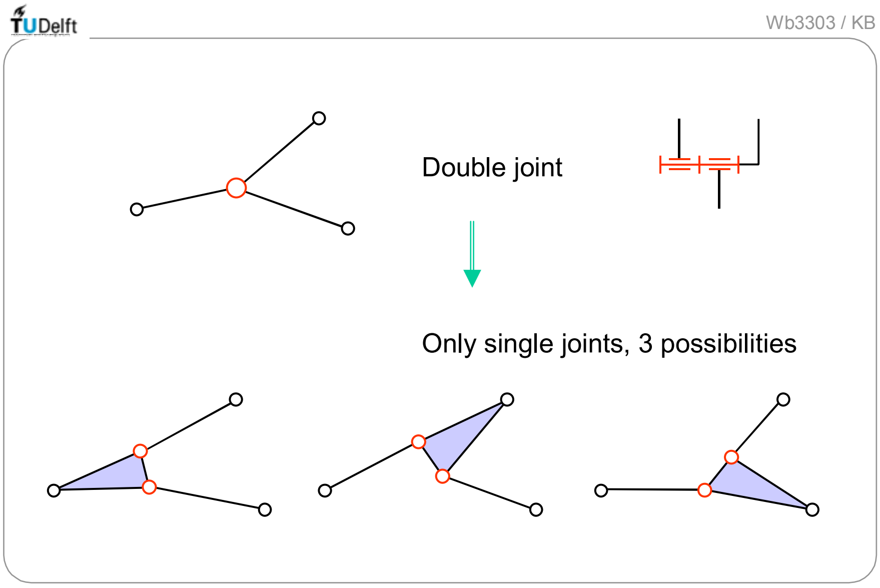

Fig. 2.2.2 Double joint replacement:
1 binary link → 1 ternary link

A first question is "the degree of freedom of a kinematic chain". Observing that a planar element without any connection has three independently possibilities to move (two translations and a rotation) and that any revolute joint takes away two translation possibilities, Grübler formulated the degree of freedom of the basic chain as:

$$F = 3(n-1) - 2g_1 \qquad (2.1)$$

in which n is the number of links (including the frame) and g_1 is the number of single revolute joints. To avoid misinterpretation of this formula it must be noted that:

- a rigid sub-structure must be considered as one element (e.g. a triangle consisting of three binary links must be regarded as one ternary link),
- a point in the drawing may not be precisely the same as a single revolute joint:
  - ➢ a loose end of a link or a coupler point is not a revolute joint, and
  - ➢ a double joint, see figure 2.2.2, should be regarded "split up". Consequently this point includes two single revolute joints, while one of the three elements holds the two connections and should be regarded as a ternary link instead of a binary link.

With these assumptions the number of single revolute joints follows, instead by counting, also from:

$$2g_1 = n_1 + 2n_2 + 3n_3 + ... = \sum_{i=1}^{i_{max}} i \cdot n_i \qquad (2.2)$$

A special kind of chain is a ***closed kinematic chain***. It is characterised by the fact that all links have minimal two (revolute) joints with another link. Without further prove it is stated that, for a closed kinematic chain, the value of i_{max} (the highest amount of single revolute joints that can possibly be at one element) is determined by:

$$2 \le i_{max} \le \frac{n}{2} \qquad (2.3)$$

In case of an odd number n this value should be truncated to the nearest lower integer value.

2.3 Structure specification and configuration

The formula of Grübler offers some insight into the question "which basic kinematic chains exist with a certain degree of freedom F". The formula (2.1) can be rewritten as:

$$g_1 = \frac{1}{2}\{3(n-1) - F\} \qquad (2.4)$$

Because the number of joints must be an integer value, the number of links n must be:

- even in case that the degree of freedom F is odd, and
- odd in case that the degree of freedom F is even.

It can be concluded:

Kinematic chains with an even number of links have an odd degree of freedom

Kinematic chains with an odd number of links have an even degree of freedom

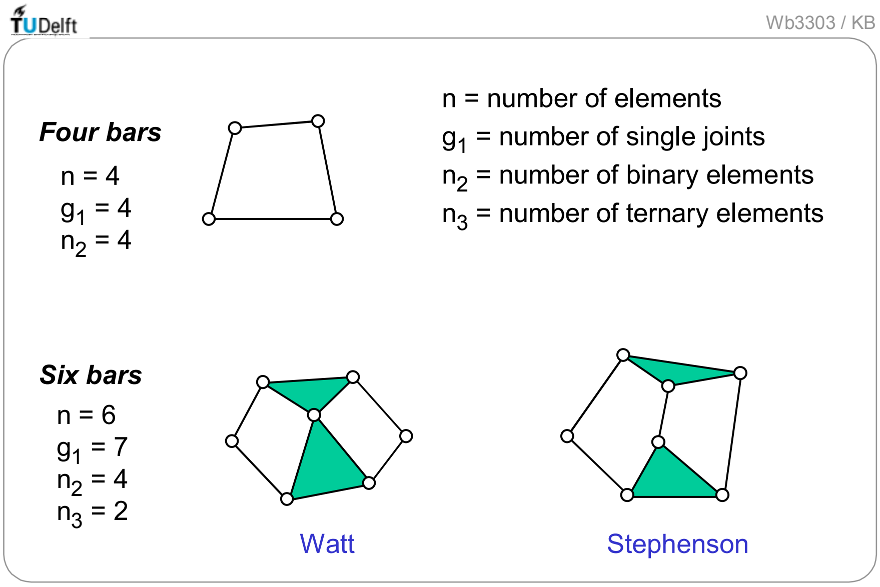

Fig. 2.3.1 *Basic kinematic chains with 4 and 6 bars*

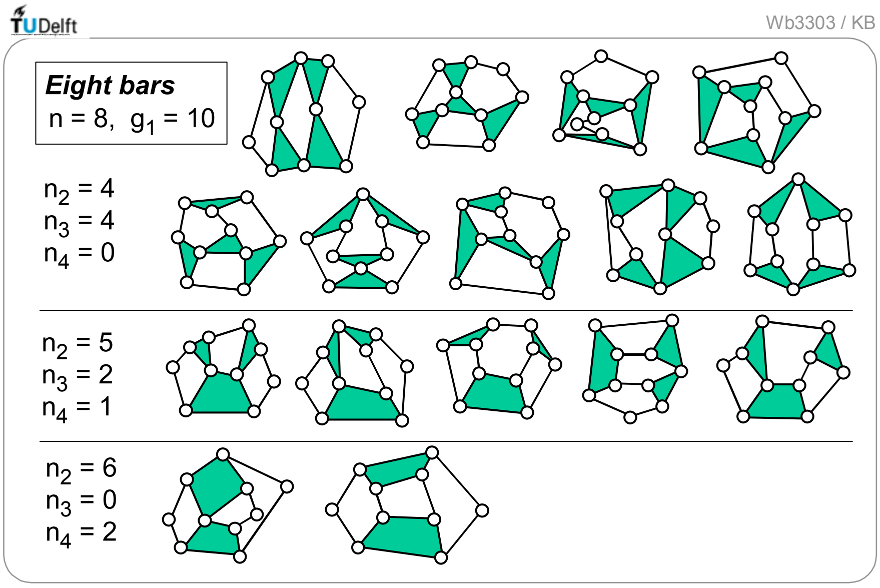

Fig. 2.3.2 *Basic kinematic chains with 8 bars and F=1*

2.3.1 Kinematic chains with one degree of freedom

With F=1 eq.(2.4) reads

$$g_1 = \frac{1}{2}\{3(n-1) - F\} = \frac{1}{2}(3n-4) \qquad (2.5)$$

Since n must be even, the values n=2,4,6,8,··· come into account.

For ***n=2 elements*** the value of g_1 will be 1. This structure contains just two bodies connected by just one hinge (like a rotor and a stator). It is typically an open chain.

The interest of this book starts with ***n=4 elements*** forming a closed kinematic chain. Eq.(2.3) shows that i_{max} =2, which means that only binary links are applicable. Eq.(2.5) shows that this chain has g_1 = 4 revolute joints. With 4 bars there is obviously only one way to make a closed chain, like drawn in figure 2.3.1.

With ***n=6 elements*** there may be both binary and ternary links according eq. (2.3). The structure relations provide equations for their amount. From equation (2.5) it follows that this chain has g_1 = 7 revolute joints.

- By definition it must be that $n = n_2 + n_3 = 6$.
- Equation (2.2) specifies that $2g_1 = 2n_2 + 3n_3 = 14$

These above mentioned two equations in n_2 and n_3 have precisely one solution: $n_2 = 4$ and $n_3 = 2$. In other words: the six-bar closed chain has one specification (four binary links and two ternary links). By systematic variation it can be found that two different configurations can be made, see fig 2.3.1 lower part. They are well known as:

- Chain of Watt (the two ternary elements are connected by a joint), and the
- Chain of Stephenson (no connection between the two ternary links).

For ***n=8 elements*** the following can be derived.

- From (2.3): $i_{max} = 4$, so the kinematic chains may contain binary, ternary and quaternary links.
- From (2.2): the number of revolute joints $g_1 = 10$.
- By definition it must be that $n = n_2 + n_3 + n_4$, and
- Equation (2.2) specifies that $2g_1 = 2n_2 + 3n_3 + 4n_4 = 20$.

The last mentioned two equations have three unknowns (n_2, n_3 and n_4), so there is more than one solution possible. The following three specifications exist, ordered to the number of quaternary elements:

- $n_4 = 0,\ n_3 = 4,\ n_2 = 4$
- $n_4 = 1,\ n_3 = 2,\ n_2 = 5$
- $n_4 = 2,\ n_3 = 0,\ n_2 = 6$

According to Hain [2.2] 16 different configurations can be made, see fig. 2.3.2.

It will be clear that kinematic chains with 10, 12 and more elements will have numerous different specifications and configurations. According to Crossley [2.3] the kinematic chain with 10 elements has 7 specifications and 230 different chain configurations. According to Kiper [2.4] the kinematic chain with 12 elements has 15 specifications and 6856 different configurations.

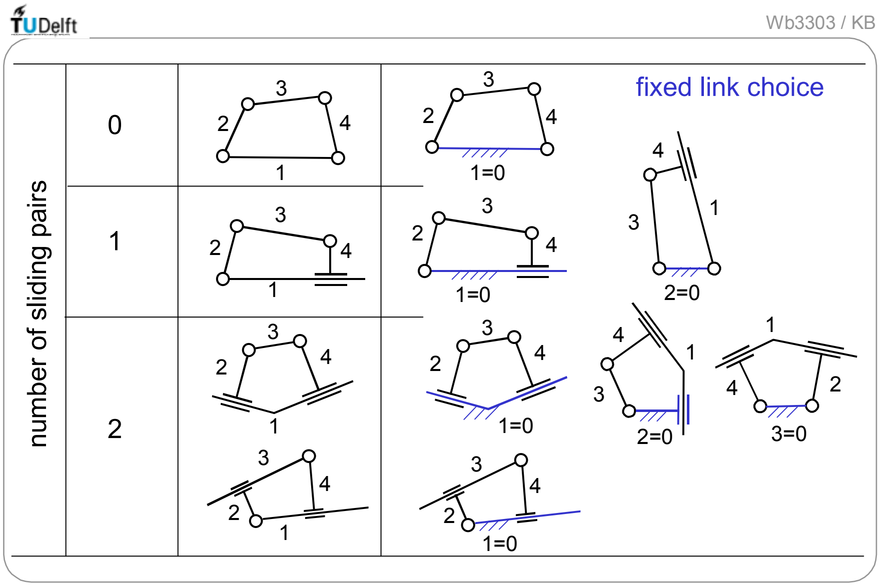

Fig. 2.4.1 *Kinematic configurations based on the four-bar chain, revolute pair → sliding pair replacement*

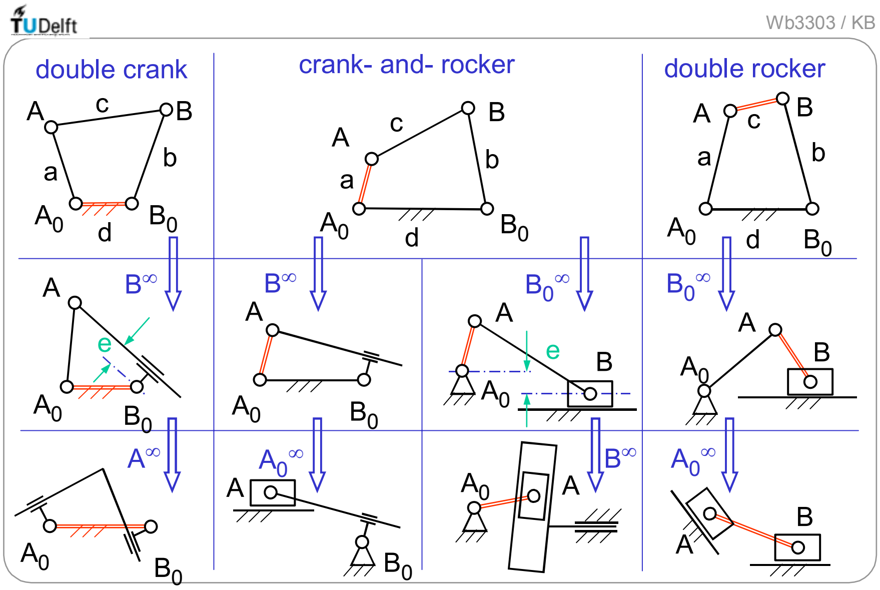

Fig. 2.4.2 *Different mechanisms with respect to motion type*
full revolution acc. Grashof

2.3.2 Kinematic chains with two degrees of freedom

In this chapter it was detected already that such chains must have an odd number of elements and, according to eq. (2.3), the number of single joints g_1 is

$$g_1 = \frac{1}{2}\{3(n-1) - F\} = \frac{1}{2}(3n-5) \qquad (2.6)$$

For ***n=3 elements*** the number of joints is then 2. Only an open chain can be found, see figure 2.6.1 upper left (one specification, one configuration).

For ***n=5 elements*** the number of joints is 5. A closed chain can be made with only binary elements, see figure 2.6.2 upper left (one specification, one configuration).

2.3.3 Kinematic chains with three degrees of freedom

The chain must have an even number of elements and the number of single joints g_1 is

$$g_1 = \frac{1}{2}\{3(n-1) - F\} = \frac{1}{2}(3n-6) \qquad (2.7)$$

For ***n=4 elements*** the number of joints is 3. Only an open chain exists, see fig. 2.6.3 left.

For ***n=6 elements*** the number of joints is 6. A closed chain can be made, but only with binary links. This does not directly follow from the structure equation (2.2), but is a conclusion from attempts to apply ternary elements. There is obviously only one specification and one configuration possible.

For ***n=8 elements*** the number of joints is 9. There are two closed chains possible, see fig. 2.6.4 below.

In chapter 2.6 the application of multi-degree-of-freedom mechanisms will be considered further.

2.4 Topological variants of a kinematic chain (4 bar mechanisms)

Two new structural effects will be introduced in this chapter and they will be applied first to the four-bar kinematic chain.
Until now only revolute joints were considered to be present in the kinematic chain. A sliding pair can however simply replace a single revolute joint. This does in general not change the degree of freedom of the chain. By doing this systematically it is thus possible to derive new kinematic chains from the basic chain, see fig. 2.4.1, left column. The newly derived chains are sometimes called the topological variants of the basic chain. (Note: some combinations of joint replacement lead to exceptional configurations; they will not be considered here further.)
The second effect is the specification of the fixed link. By definition a kinematic chain may be called then a mechanism. The fixed link can be chosen such that all possible different mechanism types come into account.
Both structural effects together yield a matrix of different mechanism types, see figure 2.4.1 for the four-bar chain.

As can be done specifically for the four bar mechanism a third structural effect is to distinguish the type of motion (transfer function). Basically the question is whether a revolute joint can make a full revolution or not. This is obviously dependent on the length of the links. Grashof specified this condition as follows:

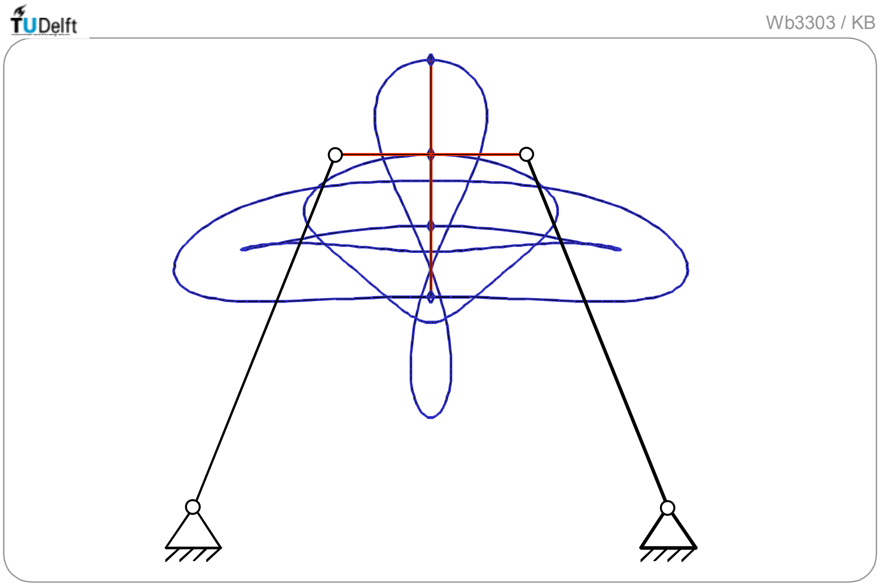

Fig. 2.4.3 Examples of coupler curves generated by a four-bar linkage (symmetrical curves)

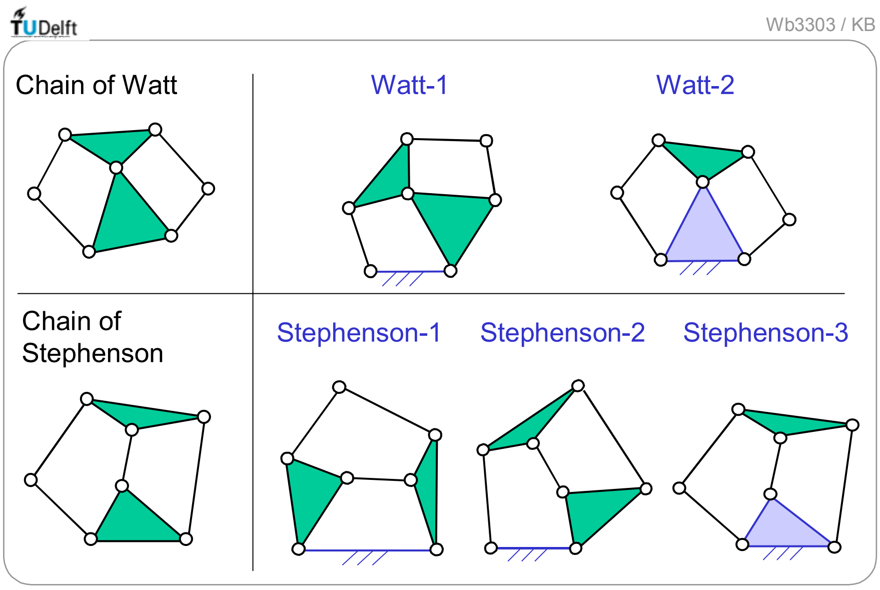

Fig. 2.5.1 Different six-bar mechanisms by fixed link choice

Law of Grashof:

The shortest link of a four-bar mechanism can make a full revolution relative to the other links if the sum of the shortest and the largest link is less then or equal to the sum of the lengths of the other two links.

In formula: $\ell_{min} + \ell_{max} \leq \ell_3 + \ell_4$ (2.7)

A full revolution is important when a periodic transfer function is required. This is usually the case for a machine concept with a central driving shaft.
When the Grashof condition is true, there are still three possibilities, see fig. 2.4.2:

- The shortest link is pin-jointed to the frame (crank-and-rocker mechanism),
- The shortest link is the coupler (double rocker mechanism),
- The shortest link is the frame. In this case all other links make a full revolution relative to the frame (double crank mechanism)

The names of these mechanisms are thus determined by the two links adjacent to the frame.
The topological variants of the four-bar linkage have names in literature. Most types have a shortest link for which the Grashof condition is valid, as can be listed in the following table.

nr	name	Grashof condition	remark
1	Double crank	yes	Frame is shortest link
2	Crank-and-rocker	yes	
3	Double rocker	yes	Coupler is shortest link
4a	Inverted crank-slider (rotating)	yes	Frame is shortest link (crank angle is input)
4b	Inverted crank-slider (rotating)	yes	Frame is the shortest link (slider angle is input)
5	Inverted crank-slider (oscillating)	yes	
6	Crank-slider	yes	
7	Rocker-and-slider	yes	Coupler is shortest link
8	Double inverted slider	yes	Frame is shortest link
9	Slider-inverted slider	no	
10	Scotch yoke	yes	Also known as sine generator
11	Double slider	yes	Also known as elliptical motion

The replacement of a revolute joint by a slider pair can be understood also as “the connection point lies in infinity”. Consequently the two adjacent bars have infinite length. Their difference remains finite, see the measure e in fig. 2.4.2, sections 5, 6 etc. (often referred to as eccentricity). Using the eccentricity the Grashof condition of the crank-slider and the inverted crank-slider can be written as

$$a + |e| \leq c \tag{2.8}$$

General remarks about the application of four-bar mechanisms with a driven crank.

- They generate transfer functions with one maximum and one minimum (no pilgrim step, no intermediate dwell).
- The rocker moves within a region of 180° relative to the frame.
- The coupler curves (path of a point in the coupler plane) have a great variety in shape. They can be symmetrical (see fig. 2.4.3 as an example) or non-symmetrical, they can have cusps, approximated straight parts, double points etc.
- The kinematic transfer quality is expressed by the pressure angle. This is the angle between the force direction at point B (along the coupler) and the motion direction of point B (perpendicular to the rocker or along the slider), see fig.2.4.2.

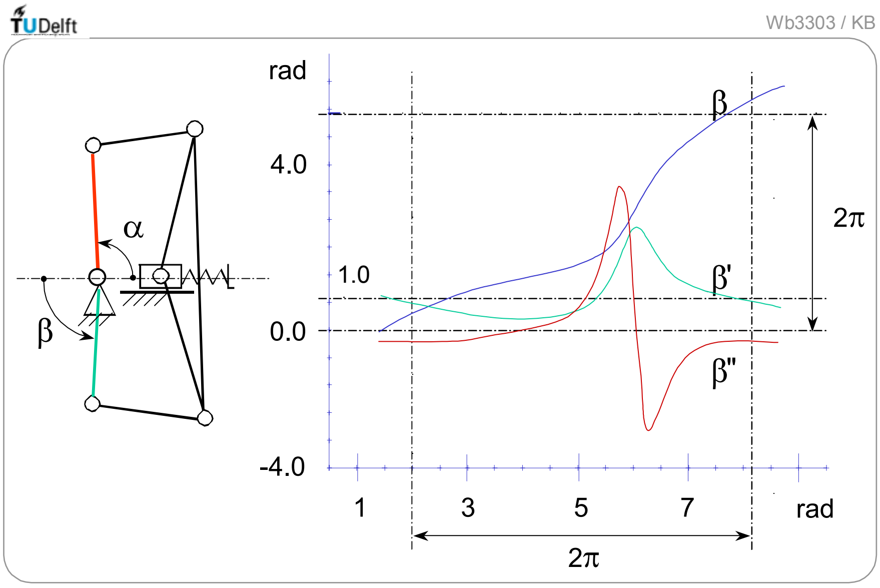

Fig. 2.5.2 *Watt-2 mechanism used as a series connection of two double crank mechanisms (adjustable frame link).*

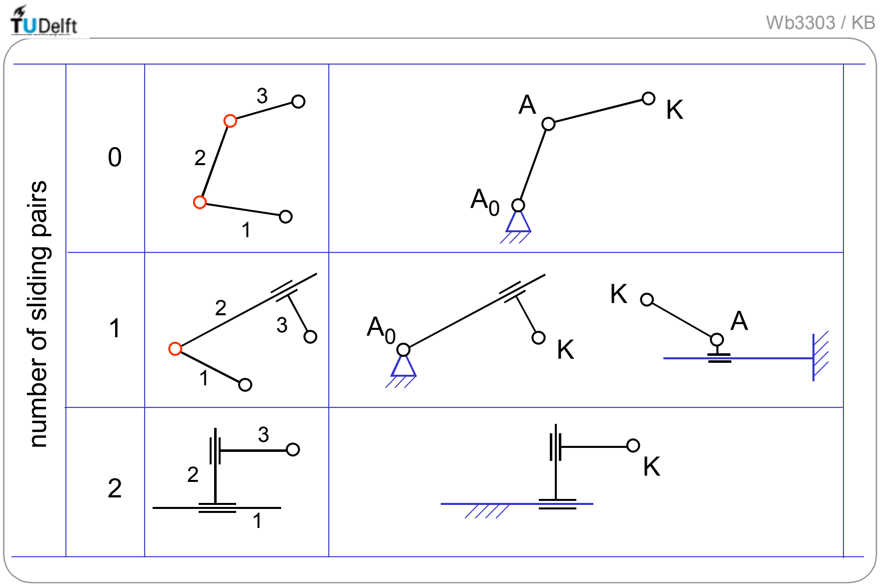

Fig. 2.6.1 *Kinematic configurations with F=2 based on the open chain with 3 bars*

When a mechanism with one or more sliding pairs is to be investigated for its degree of freedom, the reverse joint replacement can be helpful. The sliding pairs can simply be replaced by revolute pairs, this will in general not affect the degree of freedom. For the replacing mechanism, having the basic kinematic chain with only single revolute joints, the Grübler formula (2.1) can be applied to find out the degree of freedom. The different topologies of the four-bar mechanism give the opportunity to exercise this idea.

2.5 Topological variants of the 6-bar chain

The two possible closed kinematic chains with six bars and degree of freedom one (chain of Watt and chain of Stephenson) can have different choices for their fixed link, see fig. 2.5.1. In literature they are known as Watt-1,2 and Stephenson-1,2,3 mechanism.
In principle all these mechanisms have topological variants when one or more revolute joints will be replaced by a sliding pair. It is beyond the scope of this book to present here all perturbations.
An interesting practical application is the Watt-2 mechanism that can be considered as a series connection of two four-bar mechanisms.
Series connection is a principle to modify a motion (transfer function). When a single four-bar mechanism is not an accurate solution, it can be tried to drive the input link non-uniformly. Figure 2.5.2 shows an attractive implementation of this idea. The driving link and the driven link have a double revolute joint with the frame, providing an input shaft (angle α) and an output shaft (angle β) that are inline. The third frame joint is placed on a slider that can be adjusted to vary the effect of non-uniformity. When this third frame point coincides with the two others, then the transfer function is linear. So a continuous adjustable nonlinearity can be achieved with this mechanism.
(Note. When the adjustable frame point is modelled really with a slider, this mechanism should be considered to have two degrees of freedom and seven links).

2.6 Mechanisms with 2 and 3 degrees of freedom

In chapter 2.3 the basic kinematic chains with two and three degrees of freedom were already presented. As done for the four-bar chain the following two structural effects will be applied now.

- Replacement of a revolute joint by a slider pair and
- Choice for the frame link

For the (open) chain with 3 links the resulting mechanism types are depicted in figure 2.6.1. Assuming that the middle bar is a trivial choice for the fixed link, there exist four different topologies (right column in the figure).
For the (closed) chain with five links 16 different mechanism types can be found, see fig. 2.6.2
A mechanism with 2 DOF can for instance be used to decompose a desired path (in the Cartesian space) into two transfer functions, which have their output link adjacent to the frame. Referring to the machine concept of fig. 1.5.3 these mechanism types can be used as a 2 DOF manipulator. The term *parallel manipulator* is used when all driven links have a (servo) motor directly connected to the frame. In case that the two transfer functions are generated by two other mechanisms (designed as function generators) the term *parallel connection* of mechanisms is used.
A second important application of 2 DOF mechanisms is adjustable motion. One of the DOF's is used to adjust a certain fixed measure (possibly only when the mechanism does not run) to vary the motion generated by driving the other input. Figure 2.5.2 is an example of such an application.

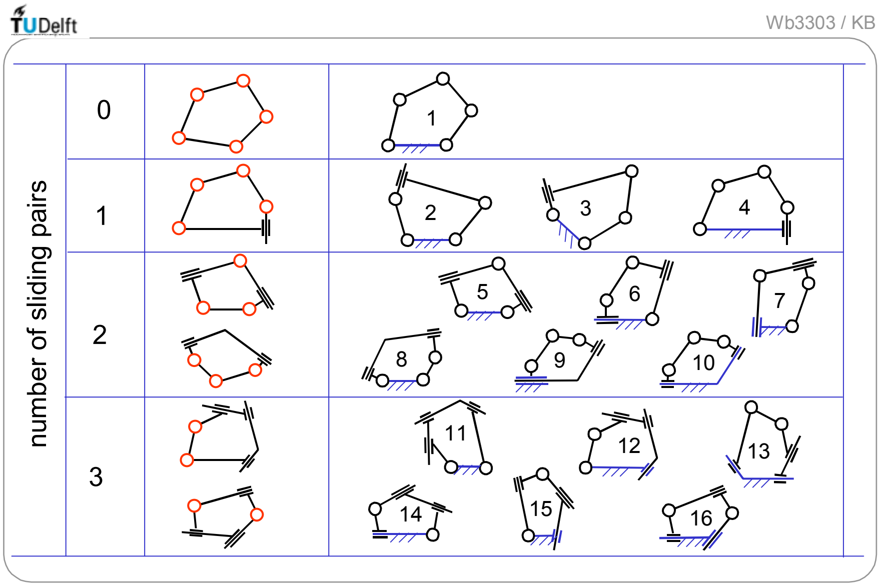

Fig. 2.6.2 Kinematic configurations with F=2 based on the closed chain with 5 bars

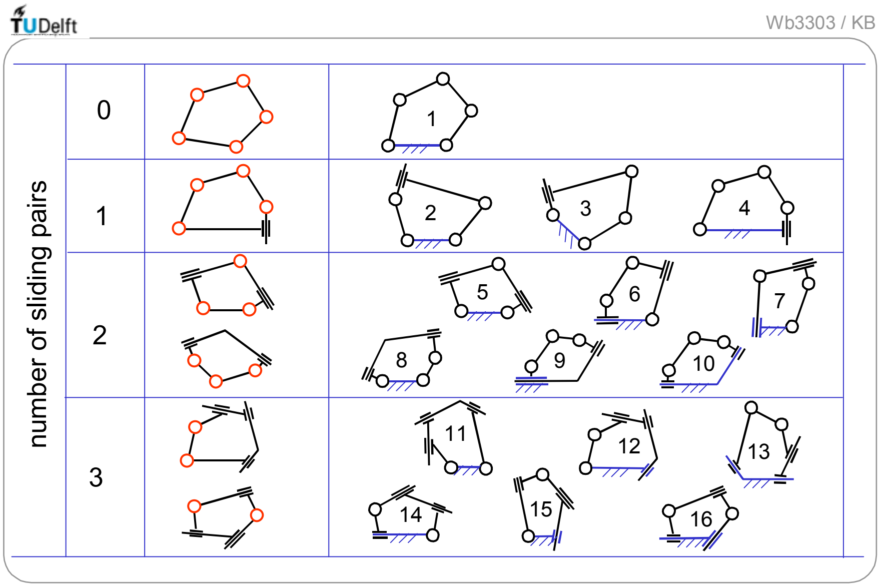

Fig. 2.6.2 Kinematic configurations with F=2 based on the closed chain with 5 bars

Three DOF mechanisms are mainly used as a planar manipulator. The three motion possibilities of a moving plane can then be controlled parallel. The eight-bar chain of fig. 2.6.4, left below, offers the possibility to place the three driving motors at the frame link (assumed that a ternary link is chosen as the frame). The other ternary link is then the plane to manipulate. A comparable manipulator is based on the open four-bar chain of figure 2.6.3. To be able to drive all three DOF's relative to the frame, parallelograms must be added to this configuration (one for link AB and two for link BK). Now this manipulator will have 10 links.

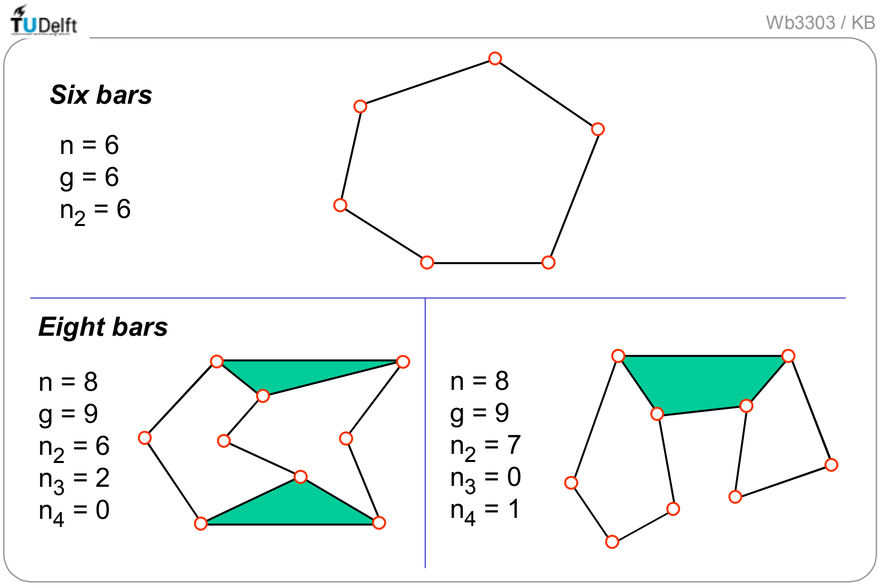

Fig. 2.6.4 Closed chains with F=3

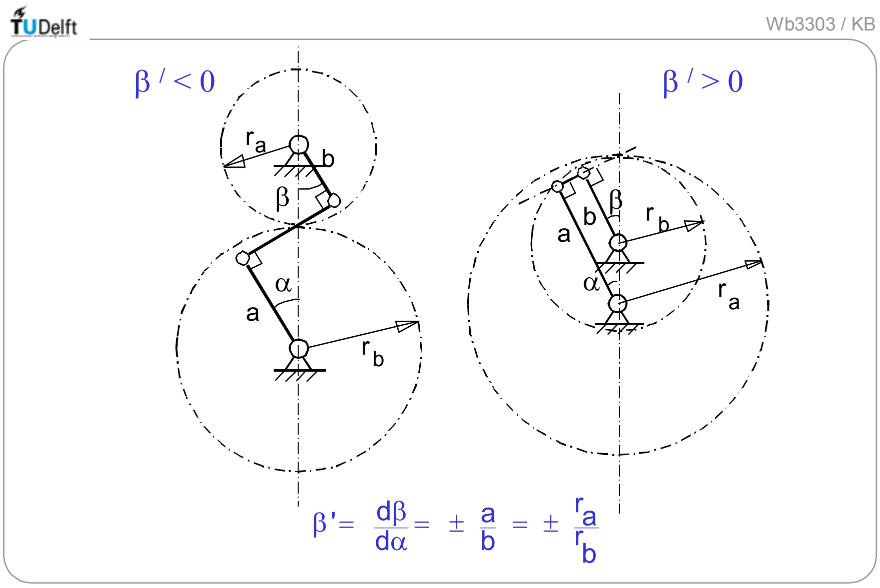

Fig. 2.7.1 Instantaneous equivalence of a gear pair and a four-bar linkage

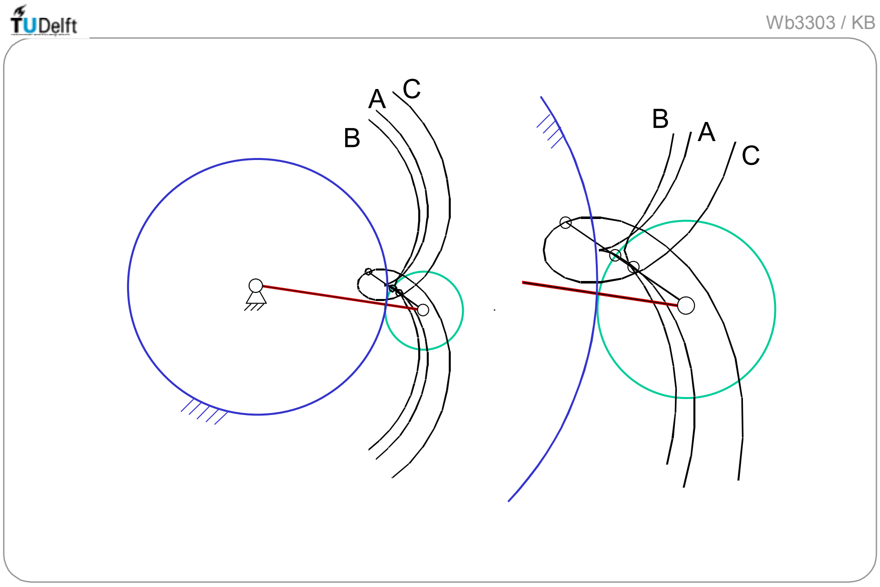

Fig. 2.7.2 Epi-cycloids, pointed (A), waved (B) and curled (C)

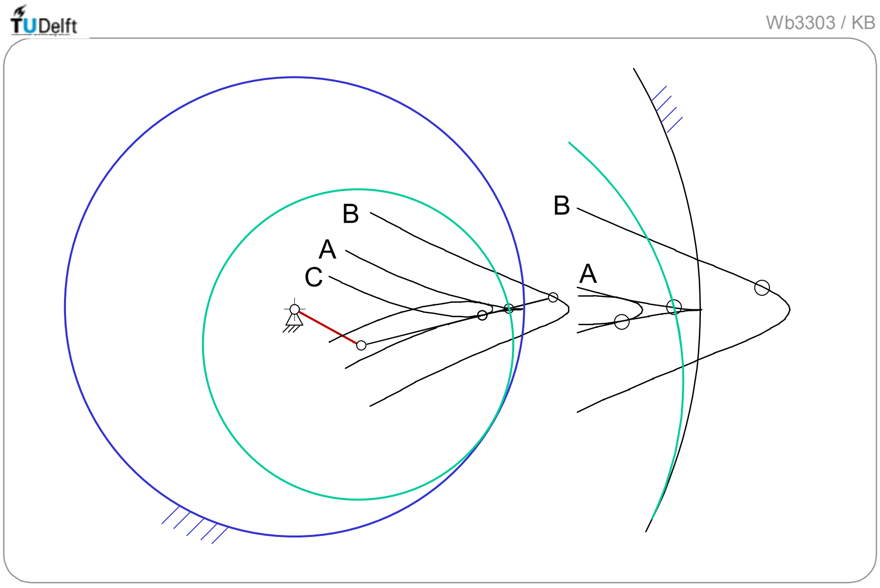

Fig. 2.7.3 Hypo-cycloids, pointed (A), waved (B) and curled (C)

2.7 Mechanisms with gears or belts

A gear pair consists of three elements: the two gears and the connecting rod between the two centre points of the gears. Together they form a (sub) chain with one degree of freedom. An equivalent structure has the four-bar chain as depicted in fig. 2.7.1. The transfer of the contact force between the gear teeth is taken over by the coupler. This equivalent structure is only valid in or near the position as drawn, but the intention is here in the first place to find a way to apply the Grübler formula (2.1) to calculate the degree of freedom. The replacement trick to be done before Grübler can be applied reads then as follows.

- Add a revolute joint to each of the gears (g_1 becomes two higher).
- Add a binary link between the two extra joints (n becomes one higher).
- Neglect the shape of the gears.

The same replacement idea can be used when two sprocket wheels and a belt are present. When a gear-and-rack appears in the mechanism, the same trick can be done. Note that the rack slides relative to the connecting element. This sliding joint can be replaced by a revolute joint anyhow.

An interesting idea is to reverse the replacement trick. This reversing includes the following. In any kinematic chain where a binary element is present, this link can be replaced by a pair of gears. This will decrease the number of elements n by one and the number of revolute joints by two, but a new configuration will be the result. By doing this systematically a great variety of new mechanism types can be derived. Two examples concerning the six-bar basic chain are presented in figures 2.7.6 and 2.7.8. There is no objection to apply this reverse replacement to topological variants with sliding pairs. In that case the slider can become teeth and will thus be a rack. It is beyond the scope of this book to go in detail here too much.

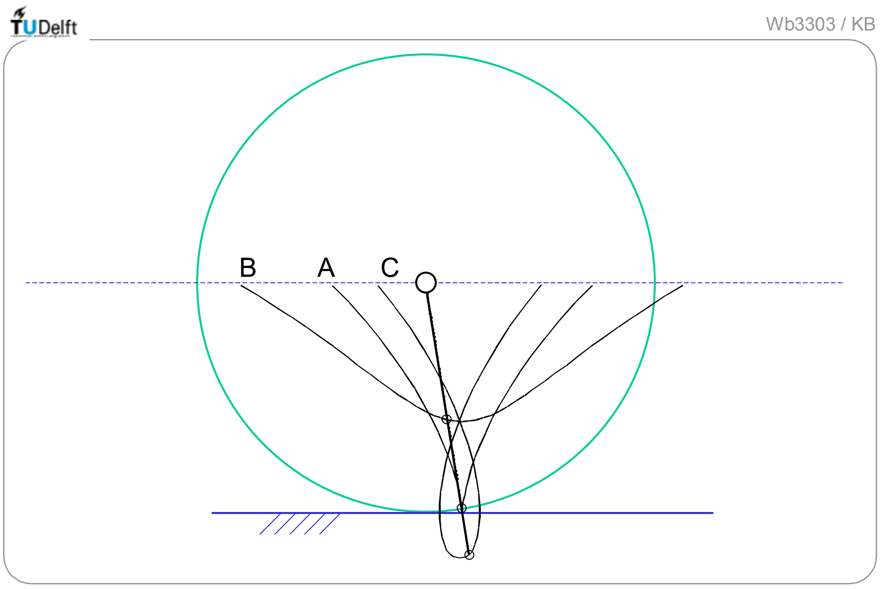

Fig. 2.7.4 Ortho-cycloids, pointed (A), waved (B) and curled (C)

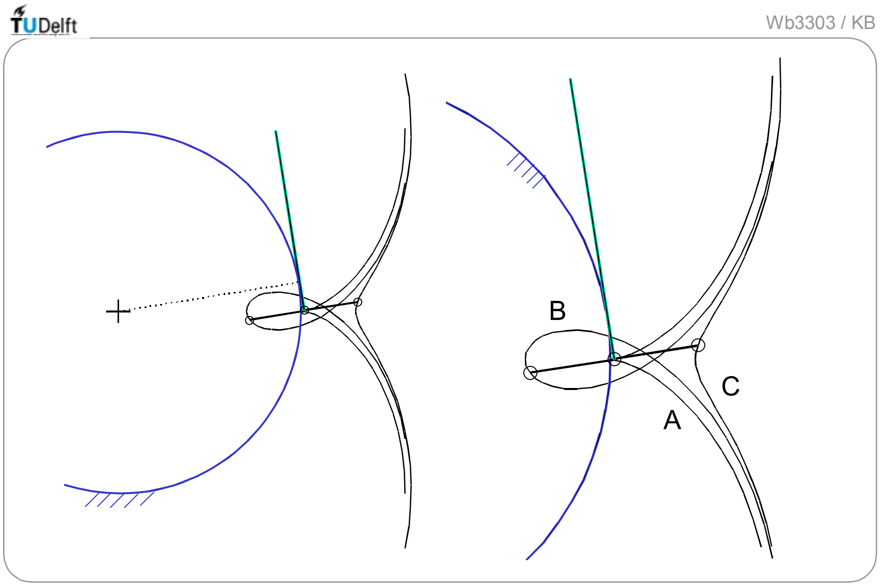

Fig. 2.7.5 Circle-involutes, pointed (A), waved (B) and curled (C)

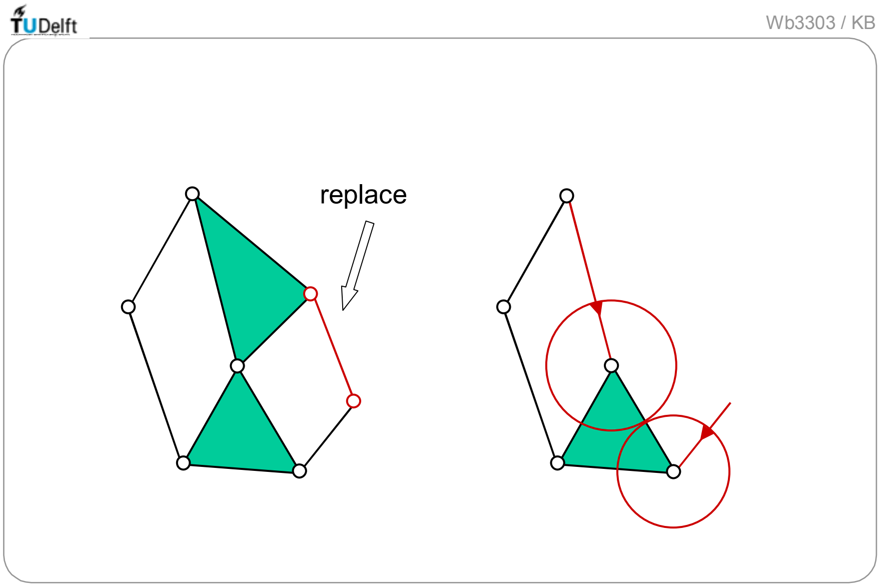

Fig. 2.7.6 Replacement of a binary link in a four-bar chain by a pair of gears (example with chain of Watt)

Mechanisms with gears have special properties, some of them will be mentioned here.
With one gear as the fixed link the points of the other gear generate a so-called trochoid. As depicted in figures 2.7.2 through 2.7.5 they can be distinguished into the following types:

- Epi-cycloid (two gears with outer contact)
- Hypo-cycloid (two gears with inner contact)
- Ortho-cycloid (gear-and-rack, the rack is the frame)
- Circle-involute (gear-and-rack, the gear is the frame)

The epi- and hypo-cycloids can be closed curves. The shape of all types can be either waved or curled (with a double point) or pointed (with a cusp). The characteristic properties of the trochoids are often a reason for their application.
A second characteristic property can be found when the connecting rod in fig 2.7.6 is taken as the driving element (crank), while the gear element having one joint (the output element) is concentric with the fixed point. In other words the ternary element reduces to a binary element with one double joint. An example of such a mechanism, having one slider pair, is depicted in figure 2.7.7. Now a superposition of two motions will be the result:

- Rotation with the crank angle α and
- Oscillating with the non-uniform motion of transfer function $\gamma(\alpha)$, but multiplied with the transmission constant i of the gear pair.

The structure of the combined function is then:

$$\delta(\alpha) = \alpha + i \cdot \gamma(\alpha) \qquad (2.9)$$

The motion type of the transfer function is thus continuous. Essentially different from the transfer function generated by a four-bar double crank mechanism (fig. 2.5.2) is that the non-uniformity can be chosen freely by the gear parameter i. It is very well possible then to create a transfer function with an intermediate dwell or a pilgrim step.

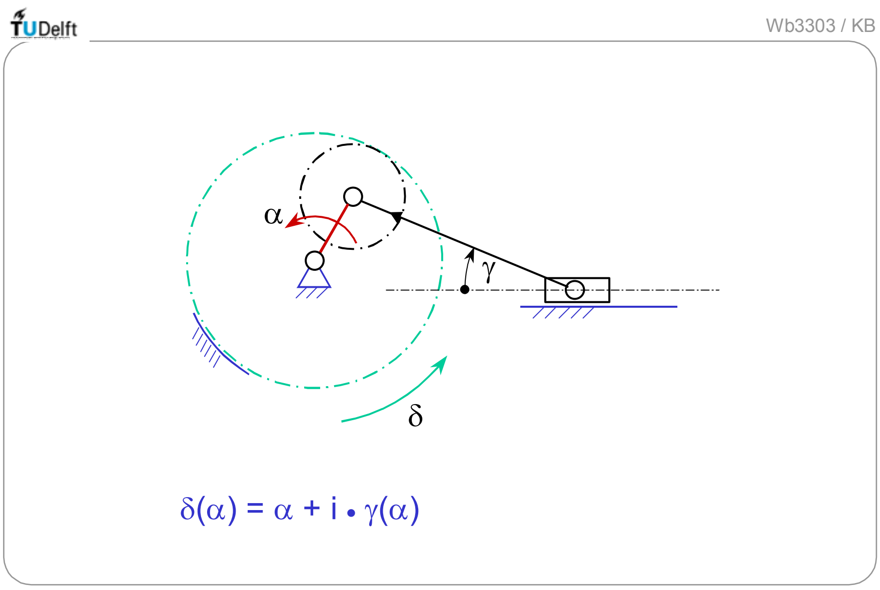

Fig. 2.7.7 Superposition of transfer functions in the Watt-chain: the basis idea for step-dwell-step motion

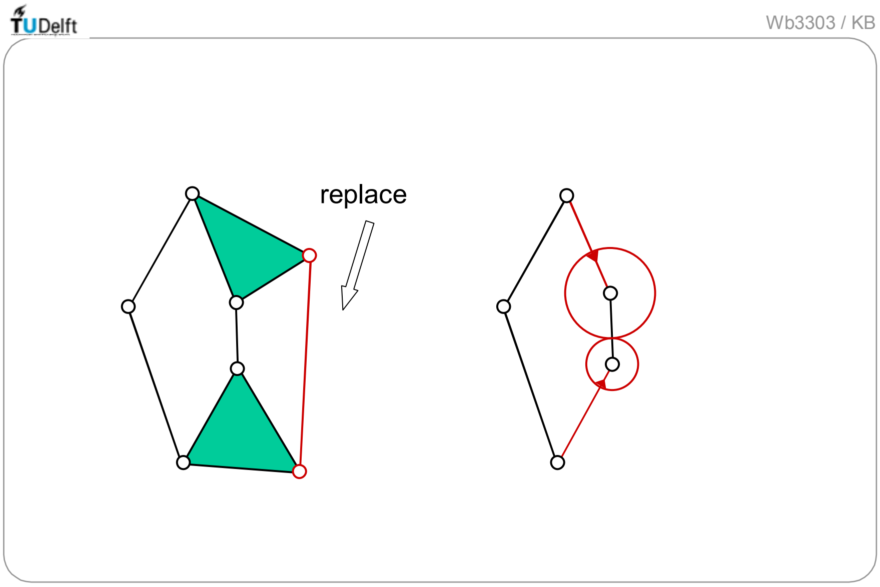

Fig. 2.7.8 Replacement of a binary link in a four-bar chain by a pair of gears (example with chain of Stephenson)

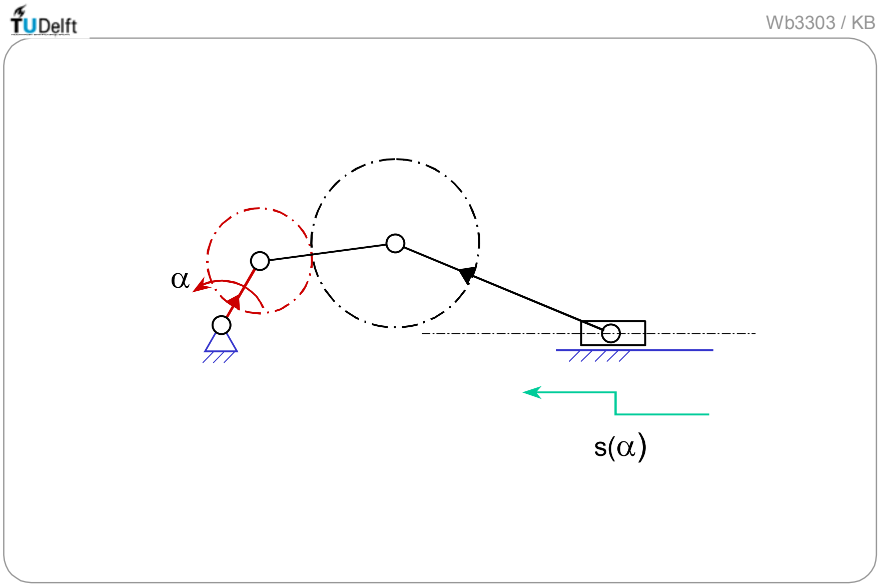

Fig. 2.7.9 Non-periodic continuous motion in the Stephenson chain

A third characteristic property can be detected looking at figures 2.7.8 and 2.7.9. It can be recognised easily that it is very well possible to have non-periodic transfer functions, even when there is a crank (an element that can make a full revolution).

2.8 Cam mechanisms

Consider two elements, each having a certain outer shape (profile), that are each pin-jointed to a connecting rod. As long as the surfaces do not touch, the degree of freedom is two. When they touch, one motion possibility is eliminated and a (sub)-chain with one degree of freedom remains, see figure 2.8.1. This type of connection is called a “higher pair”, since it allows more than one relative motion (here: both rolling and sliding of the surfaces). Application of the Grübler formula needs again a replacement trick. Here it can be found by taking into account the centres of curvature of both shapes at the touching point. A connecting rod can be thought between these centres, which replaces instantaneously the shapes on the profile elements. The sub-chain becomes then a four-bar chain. The replacement trick is thus precisely the same as for the pair of gears.

In practice cam mechanisms are used for their ability to generate any desired transfer function. Their speciality is that they can generate easily a long (finite) dwell. This will happen when the driving link has a circular shape concentric with the driving shaft. The information concerning the transfer function can be given totally to one of the profile elements, which is called then “the cam”. The other element can be given a simple (cheap) curve like a flat shape, a point or a circle. The latter curve can then also be a roller, which can be pin-jointed to the follower element. Now sliding surfaces are avoided.

Such contact between two profiles always assumes that there is either a force (e.g. coming from a spring) or a second contact (coming from a second higher pair) to maintain the contact. Terminology: *force-closed pair* and *form-closed pair*.

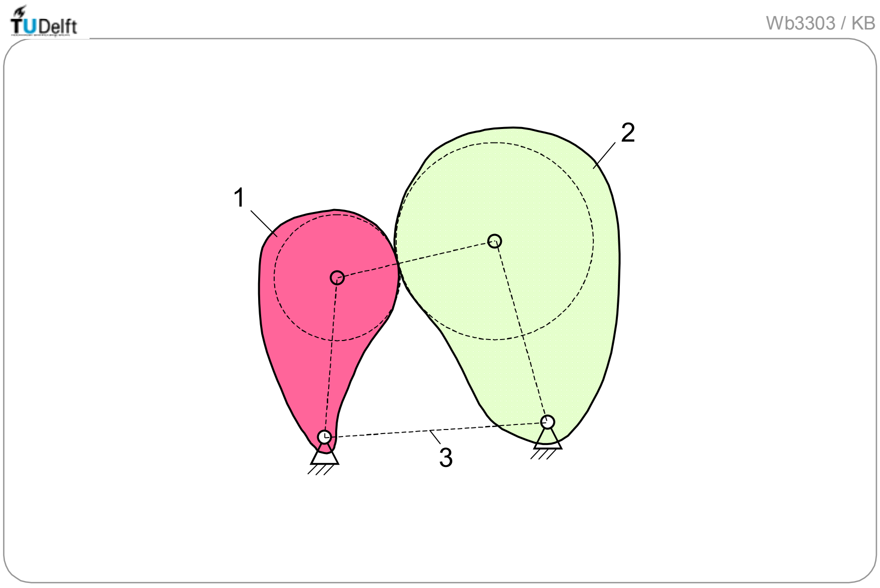

Fig. 2.8.1 *Instantaneous equivalence of three binary links and two curves with sliding and rolling contact*

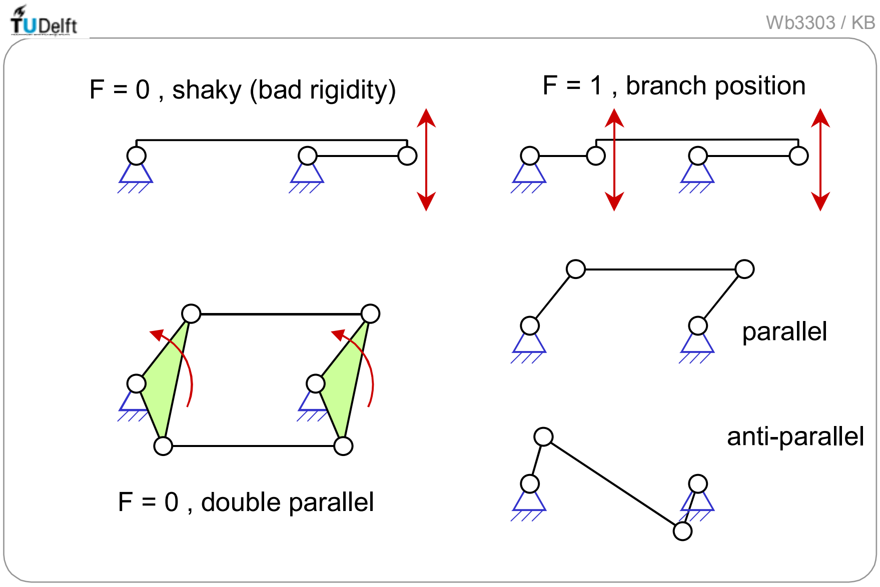

Fig. 2.9.1 Special dimensions may cause exceptions of Grübler's law

2.9 Special configurations and formula of Grübler

The structure formula (2.1) through (2.3) are valid for general dimensions of the elements. For special dimension choices there may exist special configurations that need special treatment.

Examples:

- Structures with *poor rigidity* (shaky structure). Three bars that are pin-jointed normally form a rigid structure and should be regarded as one (ternary) element. In case that one bar length is the same as the sum of the other two lengths, the structure can still be assembled but the rigidity is poor, see fig. 2.9.1 upper left. According to Grübler the degree of freedom is zero, but a connecting point can infinitesimally move perpendicular to the bar without introducing internal forces in the bars. It seems to be that the degree of freedom is one higher.
- Structures with a *branch*. The parallelogram (F=1) is such a structure: a four-bar linkage for which all bars can coincide with one line. In the folded position the rigidity is poor (the same as the previous example) and it seems that F=2. From this position the mechanism has a branch to either the parallel or the anti-parallel configuration, see fig. 2.9.1 right part.
- *Overdetermined structures*. To avoid the branching problem of the previous example sometimes a second parallelogram is added, the new cranks preferably perpendicular to the original cranks. According to Grübler this 5-bar chain with 6 single revolute joints has F=0 degrees of freedom. Obviously this mechanism can move with degree of freedom one. The special dimensions cause F to be one higher. The structure is however overdetermined in each position. Such mechanisms can move only when the accuracy of the dimensions is perfect (with respect to backlash, deformations etc).

It will be clear that such special configurations may also be present in sub-chains. It is therefore obvious that the Grübler formula and the other structure formulas must be applied with care.

2.10 References

[2.1]IFToMM comm.A	Abbreviations/symbols for terms in TMM. Mech. Mach. Theory, Vol.32, No.6 (1997), pp. 641-666.
[2.2]Hain, K	Analyse und Synthese der achtgliedrigen Gelenkgetriebe. VDI-Berichte Bd.5 (1955), S.81-93.
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[2.4]Kiper, G. und D. Schian	Sammlung der Grübler'schen kinematischen Ketten mit bis zu zwölf Gliedern. Fortschr.-Ber. VDI-Z., Reihe 1, Nr. 44 (1976).