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Lecture notes on

# Mechanisms (wb3303)

Part1: theory

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## *Preface*

Designers of machinery widely recognize that non-uniform motion is a difficult topic. The purpose of this course is then to enlarge understanding of both the typical opportunities and the problems involved with the application of mechanisms. Kinematics and dynamics of linkages, i.e. the analysis of motion and force behaviour of link mechanisms, plays an essential role in the design process and is the topic of this lecture.

For practical reasons the attention will be limited to planar linkages, including the types with slider pairs and rolling contact (pair of gears, pinion and rack). Structure theory (chapter 2) shows that an unlimited number of planar mechanism types can be specified which might challenge a designer for all kinds of application.

Mechanism science exists already since several centuries, originally mainly based on graphical methods for motion analysis. These methods comprise much knowledge in a compact and visual form and are therefore still valuable, not only for understanding kinematic behaviour but also for expressing relations for design (dimensional synthesis of mechanisms). Chapters 3 and 4 present a lecturer's choice of the graphical methods, enhanced by analytical treatment as suited for programming purpose.

With the introduction of the digital computer (since about 1970) numerical calculation methods became more interesting. Research activities at the TU Delft, started by Besseling and Van der Werff, resulted in a (finite) element approach, dedicated to both kinematic and dynamic analysis of linkages. Computer programs based on this approach allow a user to perform analysis of any type of mechanism, fast and accurate. Runmec is the program to be used with this lecture. Its results can be presented in graphical form, to support the designer with understanding the motion and force behaviour and thereby with his design and construction decisions. Chapters 5 (kinematics) and 8 (dynamics) deal with explanation of the theory and provide some examples of modelling mechanisms.

Extension of the FEM-approach concerns numerical optimisation (Klein Breteler). The problem is reversed: for a given output motion the best kinematic dimensions of the links, of a given type of mechanism, are to be calculated. It allows the user to concentrate on the question "which mechanism type is best suited" because it becomes easy to perform numerical experiments with different mechanisms. In chapter 6 the theory is summarized and the mental concept of optimising mechanisms is explained.

Computer controlled driving became also more interesting in the past decades. Direct application of output motion to the working element is usually not possible. The mechanism for motion transfer can sometimes, not only for constructional reasons, advantageously be of the non-uniform type. The combination "controlled drive and mechanism" requires that kinematics can inversely be performed (input and output interchanged). Chapter 7 presents the theory and examples of inverse kinematics. Planar robots are also covered by this theory.

Although the course is scheduled now in the Masters program, it can be regarded certainly as a general course for a wide group of students of the construction related faculties, like:

- Mechanical engineering (production machinery, transport equipment, industrial robots, instruments)
- Marine engineering (offshore jack-up system)
- Aerospace engineering (aircraft wing extension, collapsible solar panel)
- Industrial Design (consumer products)
- Civil engineering (drawbridge, lock door)
- Architecture (moving façade elements or roofs)

Entry requirements: the lectures "mechanics" of the BSc programs.

Part II of the lecture notes concern the user manual of the program Runmec.

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## Terminology, English – Dutch

English	Dutch	chapter
Bresse normal circle	tangentiaalcirkel	4
coil spring	spiraalveer	8
collineation axis	collineatie as	4
crank-and-rocker mechanism	kruk-slinger mechanisme	2
cuspid	spits	5
degree of freedom (DOF)	vrijheidsgraad	1,2
disc cam	nokschijf	2,3
evolute	evolute	3
fixed polode	poolbaan (vaste polhode)	3
frame motion	sleeptbeweging	3
gear (pinion)-and-rack	heugel en rondsel	3
inflection centre	buigpool	4
inflection circle	buigcirkel	4
instantaneous centre of rotation	momentane pool	3
inverted slider-crank mechanism	kruk-sleuf mechanisme	2
involute	evolvente	2
joint (=kinematic pair)	scharnier- of schuifverbinding	1,2
kinematic chain	kinematische keten	2
limit position	grensstand	5
locus	meetkundige plaats (puntenverzameling)	3,6
moving polode	poolkromme (bewegende polhode)	3
pressure angle	drukhoek	2,5
prime mover	aandrijfmotor	1
revolute pair	scharnierpunt	2
slider pair	schuifverbinding	2
slider-crank mechanism	kruk-schuif (kruk-drijfstaaf) mech.	2
transfer function	overdrachtsfunctie	1
transmission ratio	overbrengingsverhouding	1
uniform motion	eenparige beweging	1,2